11. R. O. Bell, M. Toulemonde, and P. Siffert, "Calculated temperature distribution during laser annealing in silicon and cadmium telluride," Appl. Phys., 19, No. 3, 313-319 (1979).
12. L. V. Gurvich, I. V. Veits, V. A. Medvedev, et al., Thermodynamic Properties of Individual Substances. Handbook Edition in Four Volumes, Vol. 2, Book 1 [in Russian], Nauka, Moscow (1979).
13. V. M. Glazov, S. N. Chizhevskaya, and N. N. Glagoleva, Liquid Semiconductors [in Russian], Nauka, Moscow (1967).
14. G. D. Ivlev, "Dynamics of annealing ion-doped silicon by monopulse radiation of a ruby laser," Pis'ma Zh. Tekh. Fiz., 8, No. 8, 468-472 (1982).

HEAT-CONDUCTION PROBLEM FOR A PLATE WITH A SQUARE CUT

Yu. M. Kolyano, V. I. Gromovyk, and Ya. T. Karpa

UDC 536.24.02

A method of determining the steady temperature field in a thin plate with a square cut is proposed on the basis of the theory of function continuation.

Consider a plate of thickness 2δ with a square cut, at the surface $|x_1| = b$, $|x_2| < b$; $|x_2| = b$, $|x_1| < b$ of which a heat flux $q/2\delta$ is specified, i.e.,

$$\lambda T_{,i}|_{|x_i|=b} N(x_{i\pm1}) = \mp \frac{q}{2\delta} N(x_{i\pm1}), \quad i = 1,\ 2, \tag{1}$$

where $N(x_i) = S_+(x_i + b) - S_-(x_i - b)$ are asymmetric functions of the cut; $T_{,i} = \partial T/\partial x_i$; the subscript $i \pm 1$ in Eq. (1) means that

$$i\pm1 = \begin{cases} 2, & i = 1, \\ 1, & i = 2; \end{cases}$$

at infinity the temperature is zero. The cut dimension $2b$ is commensurate with the plate thickness 2δ.

The heat-conduction equation for determining the temperature field in the given plate takes the form [1]

$$\Delta T = \varkappa^2 T, \tag{2}$$

where $\varkappa^2 = \dfrac{\alpha}{\lambda\delta}$; $\Delta = \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial x_2^2}$ is the Laplacian operator.

Introducing the function

$$\theta = TN(x_1,\ x_2), \tag{3}$$

where $N(x_1, x_2) = 1 - N(x_1)N(x_2)$, its first and second derivatives with respect to x_1, x_2 take the form

$$\begin{gathered}\theta_{,i} = T_{,i}N(x_1,\ x_2) - [T|_{x_i=-b-0}\delta_+(x_i + b) - T|_{x_i=b+0}\delta_-(x_i - b)]\, N(x_{i\pm1}),\\ \theta_{,ii} = T_{,ii}N(x_1x_2) - [T_{,i}|_{x_i=-b-0}\delta_+(x_i + b) - T_{,i}|_{x_i=b+0}\delta_-(x_i - b) +\\ + T_{x_i=-b-0}\,\delta'_+(x_i + b) - T|_{x_i=b+0}\,\delta'_-(x_i - b)]\, N(x_{i\pm1}), \quad i = 1,2,\end{gathered} \tag{4}$$

where

$$\delta_\pm(\zeta) = \frac{dS_\pm(\zeta)}{d\zeta}; \quad \delta'_\pm(\zeta) = \frac{d\delta_\pm(\zeta)}{d\zeta}.$$

Institute of Applied Problems of Mechanics and Mathematics, Academy of Sciences of the Ukrainian SSR, Lvov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 2, pp. 312-315, February, 1985. Original article submitted November 29, 1983.

Multiplying each term of Eq. (2) by $N(x_1, x_2)$ and taking account of Eqs. (3) and (4), the following equation with singular coefficients is obtained:

$$\Delta\theta - \varkappa^2\theta = -\sum_{i=1}^{2}[T|_{x_i=-b-0}\,\delta'_+(x_i + b) -$$
$$- T|_{x_i=b+0}\,\delta'_-(x_i - b) + T_{,i}|_{x_i=-b-0}\,\delta_+(x_i + b) -$$
$$- T_{,i}|_{x_i=b+0}\delta_+(x_i - b)]\,N(x_{i\pm1}). \tag{5}$$

Introducing the integral characteristic [2]

$$\vartheta = \frac{1}{2b}\int_{-b}^{b} T|_{x_1=\pm(b+0)}\,dx_2 = \frac{1}{2b}\int_{-b}^{b} T|_{x_2=\pm(b+0)}dx_1 \tag{6}$$

and taking account of the boundary conditions in Eqs. (1) and (6), Eq. (5) may be replaced by the equation

$$\Delta\theta - \varkappa^2\theta = -\sum_{i=1}^{2}\{Q[\delta_+(x_i + b) + \delta_-(x_i - b)] + \vartheta[\delta'_+(x_i + b) - \delta'_-(x_i - b)]\}\,N(x_{i\pm1}). \tag{7}$$

Using an integral Fourier transformation with respect to x_i, the solution of Eq. (7) when $\theta_i|_{|x_i|\to\infty} = 0$ is found in the form

$$\theta = \frac{1}{2\pi}\sum_{i=1}^{2}\int_{x_i-b+0}^{x_i+b-0}\left\{Q[K_0(\varkappa\beta^+_{i\pm1}) + K_0(\varkappa\beta^-_{i\pm1})] - \varkappa\vartheta\left[\frac{x_{i\pm1}+b}{\beta^+_{i\pm1}}K_1(\varkappa\beta^+_{i\pm1}) - \frac{x_{i\pm1}-b}{\beta^-_{i\pm1}}K_1(\varkappa\beta^-_{i\pm1})\right]\right\}d\zeta. \tag{8}$$

From Eq. (8), when $x_1 = b + 0$, taking account of the limit [1]

$$\lim_{x_1\to b}(x_1 - b)\frac{K_1(\varkappa\beta_1^-)}{\beta_1^-} = \pi\delta(\varkappa\zeta) \tag{9}$$

and the identity $S(x_2 + b - 0) - S(x_2 - b + 0) \equiv N(x_2)$, the following expression is obtained:

$$T|_{x_1=b+0} = \frac{1}{2\pi}\left\langle\int_0^{2b}\left\{Q[K_0(\varkappa\beta_2^+) + K_0(\varkappa\beta_2^-)] - \varkappa\vartheta\left[\frac{x_2+b}{\beta_2^+}K_1(\varkappa\beta_2^+) - \frac{x_2-b}{\beta_2^-}K_1(\varkappa\beta_2^-)\right]\right\}d\zeta +\right.$$
$$\left.+\int_{x_2-b+0}^{x_2+b-0}\left\{Q[K_0(\varkappa\sqrt{4b^2-\zeta^2}) + K_0(\varkappa|\zeta|)] - \varkappa\vartheta\frac{2b}{\sqrt{4b^2+\zeta^2}}K_1(\varkappa\sqrt{4b^2+\zeta^2})\right\}d\zeta + \pi\vartheta N(x_2)\right\rangle. \tag{10}$$

Here and above, $\beta_j^{\pm} = \sqrt{(x_j \pm b)^2 + \zeta^2}$, $j = 1, 2, i \pm 1$. Substituting Eq. (10) into (6), ϑ is determined. Hence

$$\theta^* = \frac{1}{2\pi}\int_{X_1-B}^{X_1+B}\left\{K_0(\sqrt{\mathrm{Bi}}B_2^+) + K_0(\sqrt{\mathrm{Bi}}B_2^-) - \frac{A_0\sqrt{\mathrm{Bi}}}{2}\left[\frac{(X_2+B)K_1(\sqrt{\mathrm{Bi}}B_2^+)}{B_2^+} - \frac{(X_2-B)K_1(\sqrt{\mathrm{Bi}}B_2^-)}{B_2^-}\right]\right\}d\xi +$$
$$+\frac{1}{2\pi}\int_{X_2-B}^{X_2+B}\left\{K_0(\sqrt{\mathrm{Bi}}B_1^+) + K_0(\sqrt{\mathrm{Bi}}B_1^-) - \frac{A_0\sqrt{\mathrm{Bi}}}{2}\left[\frac{(X_1+B)K_1(\sqrt{\mathrm{Bi}}B_1^+)}{B_1^+} - \frac{(X_1-B)K_1(\sqrt{\mathrm{Bi}}B_1^-)}{B_1^-}\right]\right\}d\xi, \tag{11}$$

where

$$A_0 = \left\langle\int_0^{2B}\int_0^{2B}K_0[\sqrt{\mathrm{Bi}(\xi^2+\zeta^2)}]\,d\xi d\zeta + \int_0^{2B}(2B-\zeta)\left\{K_0[\sqrt{\mathrm{Bi}(4B^2+\zeta^2)}] +\right.\right.$$

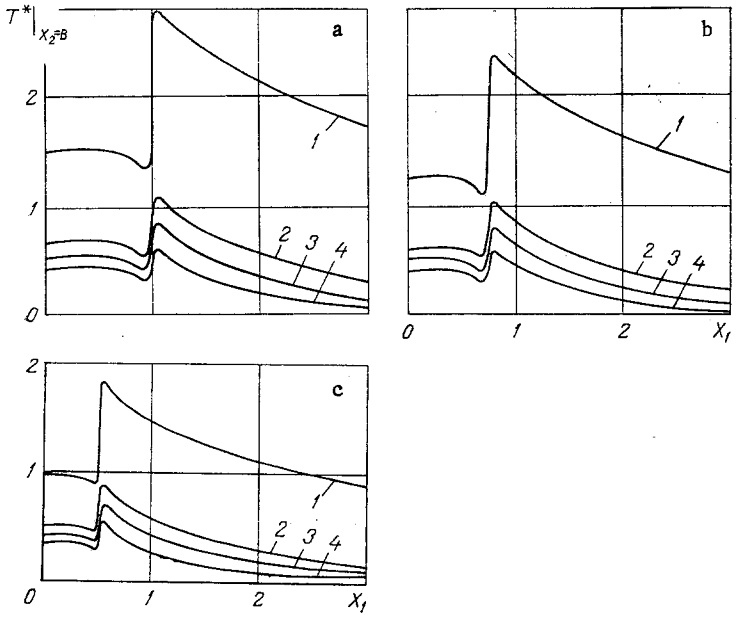

Fig. 1. Temperature distribution in plate when B = 1 (a), 0.75 (b), 0.5 (c) and Bi = 0.01 (1), 0.25 (2), 0.49 (3), 1 (4) (X_2 = B).

$$+ K_0(\sqrt{\mathrm{Bi}}\,\zeta)\Big\}d\zeta\Big\rangle\Big\langle \frac{\pi B}{2} + 2B^2\sqrt{\mathrm{Bi}}\int_0^{2B}\frac{K_1[\sqrt{\mathrm{Bi}(4B^2+\zeta^2)}]}{\sqrt{4B^2+\zeta^2}}d\zeta + \frac{1}{2}\int_0^{2B}\{K_0(\sqrt{\mathrm{Bi}}\,\zeta) - K_0[\sqrt{\mathrm{Bi}(4B^2+\zeta^2)}]\}d\zeta +$$

$$+ B[K_0(2B\sqrt{2\mathrm{Bi}}) - K_0(2B\sqrt{\mathrm{Bi}})]\Big\rangle^{-1},$$

$$B_i^{\pm} = \sqrt{(X_i \pm B)^2 + \xi^2}, \qquad X_i = \frac{x_i}{\delta}, \qquad B = \frac{b}{\delta},$$

$$\mathrm{Bi} = \frac{\alpha\delta}{\lambda}, \qquad \theta^* = \frac{4\pi\lambda\delta^2}{q}\theta, \qquad i = 1,\ 2.$$

From Eq. (11), when X_2 = B, B = 0.5, 0.75, 1, the change in dimensionless temperature field along the abscissa is calculated. It follows from Fig. 1 that the temperature decreases with increase in heat liberation from the side surfaces of the plate and with decrease in size of the quadratic cut.

NOTATION

α, heat-transfer coefficient from the side surfaces $z = \pm\delta$ of the plate; λ, thermal conductivity; $S_{\pm}(\zeta)$, asymmetric unit function [2]; $K_\nu(\zeta)$, Macdonald function of order ν; $\delta(\zeta)$, Dirac delta function; 2δ, plate thickness; Bi, Biot number.

LITERATURE CITED

1. Ya. S. Podstrigach and Yu. M. Kolyano, Nonsteady Temperature Fields and Stress in Thin Plates [in Russian], Naukova Dumka, Kiev (1972).
2. Ya. S. Podstrigach, Yu. M. Kolyano, V. I. Gromovyk, and V. L. Lozben', Thermoelasticity of Bodies with Variable Heat-Transfer Coefficients [in Russian], Naukova Dumka, Kiev (1977).